Development and Comparative Analysis of Multi-Objective Transportation Problem and its Variants

Research Plan

For Registration in Ph.D. Program

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ANNEXURE - A

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CERTIFICATE

This is to certify that this research protocol entitled **"Development and Comparative Analysis of Multi-Objective Transportation Problem and its Variants"** by Medha Sharma, submitted in fulfillment of the requirement for registration in the Ph.D. program in DEPARTMENT OF MATHEMATICS under the Faculty of Sciences of JECRC University, Jaipur, during the academic year 2023-24, is an authentic record work carried out under my guidance and supervision.

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1. Introduction

The transportation problem is a well-known optimization problem in the fields of operations research and logistics. It involves determining the most efficient and economical method of transporting commodities from various starting points to various destinations. It emerged in the disciplines of economics and logistics around the mid-20th century. Amidst World War II and the subsequent era, there arose an increasing demand for effective transportation and distribution systems to facilitate military logistics and reconstruction endeavors. The need for practical solutions, along with advancements in mathematical modeling and optimization approaches, resulted in the formalization of the transportation problem and the creation of algorithms to address it.

The task is determining the most efficient method of transporting goods from suppliers to the destination, with the goal of decreasing transportation expenses or maximizing earnings. This issue is frequently seen in the field of supply chain management and the optimization of distribution networks. The transportation issue is a specific type of linear programming problem (LPP) where commodities are moved from a group of sources to a group of destinations, taking into account the supply and demand of each source and destination. The simplex approach was introduced by Dantzig (2003). Following the publication of this method, numerous companies worldwide started adopting linear programming due to its efficacy in optimizing planning. The objective is to minimize the total transportation cost. It is occasionally referred to as the Hitchcock (1941) problem.

$$\text{Minimize / Maximize} \quad Z = \sum_{i=1}^{m} \sum_{j=}^{n} C_{ij} X_{ij}$$

$$\text{Subject To:} \quad \sum_{j=1}^{n} X_{ij} = a_i$$

$$\sum_{i=1}^{m} X_{ij} = b_j$$

$$X_{ij} \geq 0 \; \forall \; i, j$$

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j.$$

In the transportation problem, the variable X_{ij} represents the quantity of products transported from the origin i to the destination j, and the variable C_{ij} indicates the cost per unit of

transportation. The parameters a_i and b_j indicate the supply available at origin i and the demand at destination j, respectively.

Solution Methods

Several methods can be used (Dantzig (2003)) to solve the transportation problem, including:

- **Northwest Corner Method:** This is a simple heuristic method to find an initial feasible solution.
- **Least Cost Method:** Another heuristic method used to find an initial feasible
- solution.
- **Vogel's Approximation Method (VAM):** VAM is an iterative procedure such that in each step, we should find the penalties for each available row and column by taking the least cost and second least cost.
- **MODI Method:** This is an iterative method for finding an optimal solution and improving an existing feasible solution.

Key Aspects of the Transportation Problem

1. **Inputs:** The transportation problem involves the following inputs:
   - Origins: The places from where the goods are shipped.
   - Destinations: The places where the goods are delivered.
   - Costs: The cost of shipping one unit of goods from each origin to each destination.
2. **Objective:** The objective of the transportation problem is to minimize the total transportation cost while satisfying the supply and demand constraints.
3. **Constraints:**
   - Supply Constraints: The total supply from each origin cannot exceed its capacity.
   - Demand Constraints: The total demand at each destination must be met.

Decision-making has a vital role in various disciplines such as Economics, Psychology, Philosophy, Mathematics, and Statistics. The indispensability of transportation as a constituent of distribution networks must be recognized. The parameters of the TP include price, supply, and demand. Goods can be transported from one place to another using several modes of transportation in a transportation system, in order to save costs or meet delivery schedules.

The solid transportation problem (STP), commonly referred to as the three-dimensional TP, is an improved version of the well-known TP initially developed by Haley (1962). The main objective of the STP (fig.2) process is to effectively carry standardized goods from their source to their final destination by utilizing diverse transportation options in order to reduce the overall cost of transportation. The essential elements of a three-dimensional transportation plan include the availability of products at their origin points, the demand for these products at their destination points, and the capacity of various transportation modes (such as trucks, cargo aircraft, goods trains, and ships) utilized to convey the products from their origins to their destinations. The dimensions of the problem are inherently uncertain due to various factors such as machine breakdowns and labor issues in manufacturing, market conditions, road difficulties, and weather conditions during transportation. Random variables are frequently employed to describe these uncertainties, particularly those that are stochastic in nature. When formulating a practical SPP (Shortest Path Problem), it is essential to take into account the optimization of multiple objectives, including minimizing transportation time, minimizing loss during transit, and minimizing transportation costs. We need to analyze a stochastic multi-objective Shortest Path Problem (STP) based on this comprehension. The field of STP holds significant importance as a research domain, encompassing both theoretical exploration and practical implementation. A multitude of scholars in this field have made noteworthy contributions.

Many decision-making challenges in the actual world are typically depicted using several options. Multi-choice optimization methods have gained increasing importance in various fields, including technology, business, transportation, and military applications. The price indexes are the primary focus of the goal function. The variable c_{ijk} represents the cost of moving one unit of production from a specified source i to a designated destination j using a particular mode of conveyance k. In light of the rising fuel costs and other significant issues, we suggest employing a multiple-choice structure for the cost coefficients of the objective function in the transportation problem. In order to precisely measure changes in market prices for all items, it is crucial to include multiple factors that affect supply and demand. The Solid Fractional Transportation Problem (SFTP) is a comprehensive formulation that combines the characteristics of both the FTP and the STP. In the context of SFTP, every item being transported is assessed based on three attributes: the origin, the destination, and a third characteristic sometimes referred to as the "mode of transport" or "conveyance". The third feature refers to the method or mechanism utilized to transport the goods, so adding another

layer of complexity to the issue. The SFTP involves optimizing the distribution of goods from various suppliers to different clients, considering the fractional quantities and the specific transportation method for each item (Bhatia (1978)). This problem scenario occurs when objects can be divided into fractional parts and the transportation process involves many procedures, routes, or vehicles. In order to resolve the SFTP, it is necessary to consider both fractional quantities and the inherent attributes of the transportation method. Advanced optimization techniques, such as mixed-integer linear programming or specialized algorithms, are frequently employed to uncover optimal or nearly optimal solutions. The Secure File Transfer Protocol (SFTP) is widely used in several practical scenarios, such as Multi-Modal Transportation, Supply Chain including Divisible Goods, Logistics Planning, Emergency Response, and Infrastructure Development. The incorporation of fractional numbers and the mode of transport characteristic in the SFTP (Fig.1) showcases the intricacies of transportation scenarios in the actual world, leading to a more accurate and comprehensive approach to allocating and distributing resources.

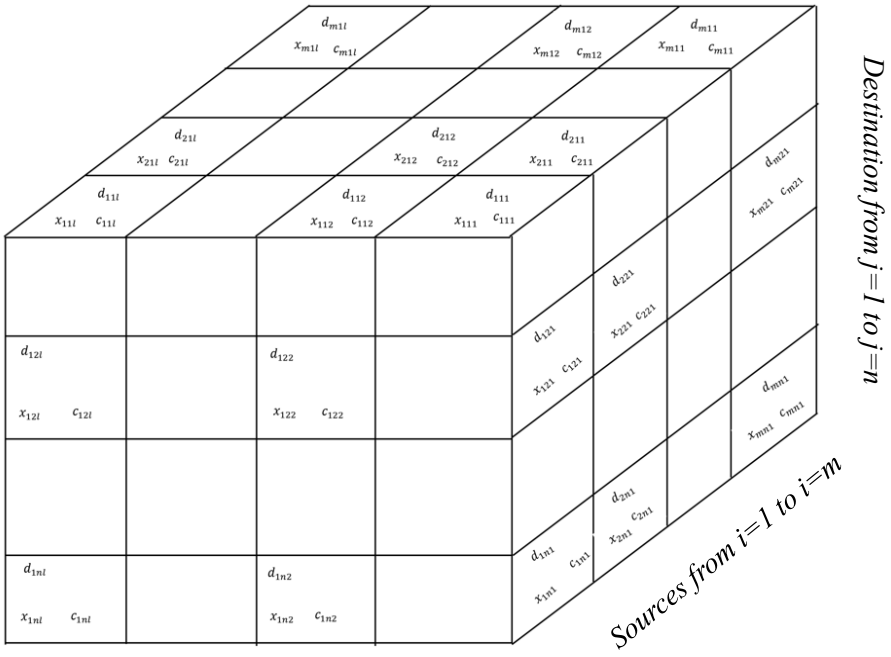


Figure 1: Tabular representation of SFTP by visualizing the cost c_{ijk}, profit d_{ijk} and decision variables x_{ijk}.

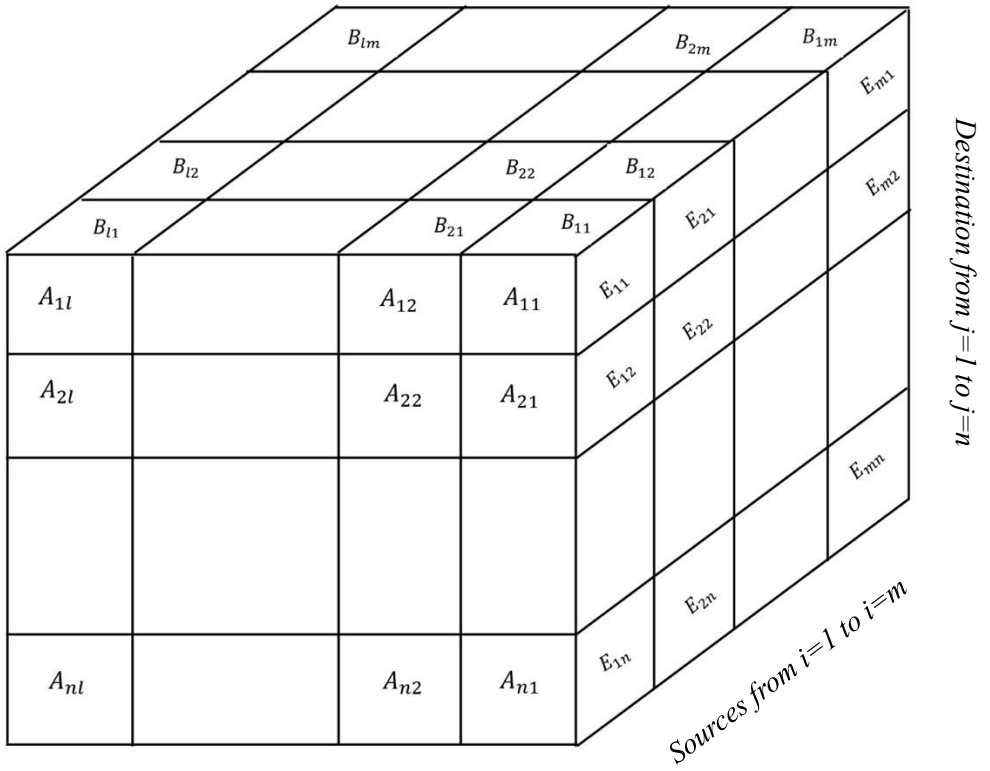


Figure 2: Tabular representation of supply (m), demand (n) and conveyance (l).

A simple possible approach for solving the SFTP requires a least of $mnl - (m-1)(n-1)(l-1)$ variables x_{ijk} (as shown in Fig.1) to have nonzero values (Haley (1962)).

Stochastic programming (SP) approaches are used when an optimization issue involves random parameters (Prékopa (1995)). This suggests that certain parameters in the model coefficients possess probability distributions that represent their knowledge with a level of uncertainty. SP commonly occurs in various practical applications in the fields of management science, engineering, and technology. These applications involve uncertain input data and models based on faulty information. Due to the swift progress of computers and modern optimization techniques in the last fifty years, there has been a growing utilization of stochastic optimization in a wide range of complex real-world decision-making scenarios. SP models have proven to be highly effective in various applications, such as supply chain management, environmental planning, telecommunications, transportation, and energy and financial resource planning.

Stochastic programming, a mathematical technique, is employed to address optimization problems that involve uncertainty. Stochastic programming focuses on the uncertainty or fluctuation of these values, in contrast to standard optimization techniques that assume fixed

values for variables. By considering a range of possible outcomes and their associated probabilities, it allows decision-makers to make informed decisions. Stochastic programming in finance can be utilized to select the optimal investment portfolio by considering different market circumstances and their respective probability. It can be utilized in supply chain management to enhance inventory levels by considering uncertain demand and supply disruptions. Stochastic programming is a powerful tool for making decisions in complex and unpredictable situations.

Mathematicians and computer scientists employ Newton's divided difference interpolation as a numerical method to estimate a function based on a set of data points. The fundamental concept is based on split differences, which involves determining the differences between connected data points. This approach facilitates the creation of a polynomial function that passes over each of the given data points. The Newton divided difference method transforms a multi-choice question into a single option question.

2. Definitions

- **Optimal solution (Dantzig (2003)):** An optimal solution is a conclusion that attains the most favorable outcome according to the specified criteria or objectives. It is akin to selecting the most advantageous solution from a range of options.
- **Pareto optimal solution (Nomani 2016):** If the objective function cannot be enhanced without decreasing one or more other objective values A feasible option is referred to as a Pareto optimum solution.
- **Ideal solution (Dantzig (2003)):** When a problem requires minimizing, the best solution is one where each objective function reaches its optimal minimum.
- **Anti-ideal solution (El-Wahed et al. (2001)):** In problems that require minimizing, the anti-ideal solution refers to a solution where each objective function reaches its highest possible value.
- **Feasible solution (Dantzig (2003)):** A set of decision variable values that fulfills every constraint of the problem is a feasible solution to an optimization problem.

3. Alternative forms of the transportation problem

The transportation problem, a classic optimization problem in operations research, has several alternative forms that extend its applicability to various real-world scenarios. Some of the alternative forms of the transportation problem include:

3.1. Multi-objective transportation problems (MOTPs)

$$\text{Minimize/Maximize } Z_t = \sum_{i=1}^{m} \sum_{j=1}^{n} C_{ij} X_{ij}$$

Subject To:

$$\sum_{j=1}^{n} X_{ij} = a_i$$

$$\sum_{i=1}^{m} X_{ij} = b_j$$

$$X_{ij} \geq 0 \ \forall \ i = 1,2 \dots m, j = 1,2 \dots n$$

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$$

In the transportation problem, the variable X_{ij} represents the quantity of products transported from the origin i to the destination j, and the variable C_{ij} indicates the cost per unit of transportation. The parameters a_i and b_j indicate the supply available at origin i and the demand at destination j, respectively (El-Wahed et al. (2001)).

3.2. Multi-objective fractional transportation problems (MOFTPs)

$$\text{Minimize/Maximize } Z_t(X_{ij}) = \frac{\sum_i^m \sum_j^n C_{ij} X_{ij}}{\sum_i^m \sum_j^n D_{ij} X_{ij}}, t = 1,2, \dots, T$$

Subject to

$$\sum_j^n X_{ij} = a_i ,$$

$$\sum_i^m X_{ij} = b_j,$$

$$X_{ij} \geq 0 , i = 1,2, \dots, m \ \ j = 1,2, \dots, n .$$

We assume that $a_i \geq 0$, $b_j \geq 0$ and C_{ij} ,$D_{ij} \geq 0$ and $\sum_i^m a_i = \sum_j^n b_j$. Let $a_i, i = 1, 2, \dots, m$ represent the supply points and $b_j, j = 1,2, \dots, n$ represent the demand sites. The goal function is defined as the quotient of the cost function, represented by the symbol C_{ij}, and the profit function, represented by the symbol D_{ij}. The variable X_{ij} represents the quantity of units to be transported from the i^{th}origin to the j^{th} destination. FTP (Fractional Transportation Problem) is a variation that permits the transportation of quantities between sources and destinations to be fractional instead of limited to whole numbers. FTP (Javaid, Jalil and Asim (2017)) is frequently used in scenarios where there is a need to convey continuous volumes of goods, such as liquid items or bulk materials. This problem expands upon the conventional transportation problem by loosening the restriction on integer values, so enabling a more optimal utilization of transportation resources. The resolution of the FTP typically entails the utilization of linear programming methods. The primary goal is to

minimize the overall transportation expenses while adhering to the limitations imposed by the availability of resources and the incomplete nature of the shipment.

3.3. Multi-objective solid transportation problems (MOSTPs)

Minimize/Maximize

$$Z_t = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l} C_{ijk}\, X_{ijk}$$

Subject to

$$\sum_{i=1}^{m} X_{ijk} = A_{jk}$$

$$\sum_{j=1}^{n} X_{ijk} = B_{ki}$$

$$\sum_{k=1}^{l} X_{ijk} = E_{ij}$$

$$\sum_{j=1}^{n} A_{jk} = \sum_{i=1}^{m} B_{ki} \;, \sum_{k=1}^{l} B_{ki} = \sum_{j=1}^{n} E_{ij} \;, \sum_{i=1}^{m} E_{ij} = \sum_{k=1}^{l} A_{jk}$$

$$\sum_{j=1}^{n}\sum_{k=1}^{l} A_{jk} = \sum_{k=1}^{l}\sum_{i=1}^{m} B_{ki} = \sum_{i=1}^{m}\sum_{j=1}^{n} E_{ij}$$

$$X_{ijk} \geq 0 \;, \forall\, i, j, k$$

The variable X_{ijk} represents the quantity of the j^{th} type of soap produced in the i^{th} plant and delivered to the k^{th} consumer. C_{ijk} represents the unit transportation cost from the i^{th} manufacturer to the k^{th} client for the j^{th} type of product. A_{jk} represents the requirement of the k^{th} client for j. B_{ki} represents the necessary quantity of units that need to be delivered from the i^{th} factory to the k^{th} consumer. E_{ij} represents the quantity of the j^{th} type of units that are now accessible at the i^{th} plant.

In order to minimize overall transportation costs while meeting supply and demand limitations, the STP optimizes the distribution of goods from suppliers to consumers using a variety of transportation modes Haley (1962). Complexities are addressed by advanced optimization methods, including linear and mixed-integer programming, as well as heuristics. STP is employed in situations involving complex logistics networks that incorporate several modes of transportation, diversified supply chains, and multiple routes or vehicles. The solutions provided by this company enable effective transportation of goods, resulting in streamlined operations and cost savings for companies that depend on complicated transportation systems to deliver products to consumers.

3.4. Multi-choice multi-objective transportation problems (MCMOTPs)

Minimize/Maximize

$$Z_t = \sum_{i=1}^{m} \sum_{j=1}^{n} \left(C_{ij}^1, C_{ij}^2, \dots, C_{ij}^T\right) X_{ij}$$

Subject to $\sum_{j=1}^{n} X_{ij} = a_i$

$$\sum_{i=1}^{m} X_{ij} = b_j$$

$$X_{ij} \geq 0, \forall\, i = 1,2 \dots m, j = 1,2 \dots n$$

The above Multi-Choice Transportation Problem (MCTP) is an expanded version of the conventional transportation problem (Roy S.K., 2016). It involves considering many transportation modes or options for delivering commodities from different origins to different destinations. Every mode of transportation may vary in terms of expenses, carrying capacity, and duration of travel. MCTP is utilized in logistics and supply chain management situations that involve many transportation modes, including trucking, rail, air, and marine shipping. This is especially pertinent in transportation networks that involve many modes of transportation, worldwide supply chains, and scenarios where a range of transportation options are necessary to accommodate different client needs and service requirements.

3.5. Multi-level transportation problems (MLTPs) (Joshi (2022))

Let's examine a T-level transportation problem that involves minimizing an objective function at each level. The T-level minimization issue can be expressed in its generic form, coupled with a set of constraints.

[Level 1]

$$\text{Minimize/Maximize}_{\bar{X}_1} \sum_{i=1}^{m} \sum_{j=1}^{n} C_{ij}^1 X_{ij}$$

[Level 2]

Where $\bar{X}_2$ solves

$$\text{Minimize/Maximize}_{\bar{X}_2} \sum_{i=1}^{m} \sum_{j=1}^{n} C_{ij}^2 X_{ij}$$

.

.

[Level T]

Where $\bar{X}_T$ solves

$$\text{Minimize/Maximize}_{\bar{X}_T} \sum_{i=1}^{m} \sum_{j=1}^{n} C_{ij}^{T} X_{ij}$$

Subject to $\sum_{j=1}^{n} X_{ij} = a_i,$

$$\sum_{i=1}^{m} X_{ij} = b_j,$$

$$X_{ij} \geq 0, i = 1,2 \dots, m \;\; j = 1,2, \dots, n$$

In the transportation problem, the variable X_{ij} represents the quantity of goods being transported from the origin i to the destination j. On the other hand, the symbol C_{ij}^{T} denotes the cost of transporting one unit of commodities. The parameters a_i and b_j indicate the supply available at origin i and the demand at destination j, respectively. The choice variables $\bar{X}_1, \bar{X}_2, \dots, \bar{X}_T$ in the above formulation are divided into distinct levels. The decision maker at the t^{th} level, where $t = 1,2 \dots, T$, has authority over the decision variable, $\bar{X}_1, \bar{X}_2, \dots, \bar{X}_T$. The level one decision maker is responsible for implementing policies and devising solutions. This high-ranking decision maker establishes its objectives or decisions and thereafter solicits the optimal solutions from each subordinate level, which are computed on an individual basis. The lower-level decision makers' decisions are subsequently submitted to and revised by the upper-level decision maker, who takes into account the total benefit to the organization.

4. Strategies for solving transportation problem

There are many Strategies for solving the TP, each with its strengths and applications:

4.1.Weighted sum method

The weighted sum strategy is commonly employed to address a Multiple Objective Linear Transportation Problem (MOLTP), resulting in a variety of outcomes depending on the provided weights. The core principle of this strategy is allocating a weight, represented as W_t, to each objective function Z_t, which is always a non-negative value. The objective is to minimize the new objective function, which is expressed as $\sum_{t=1}^{T} W_t \, Z_t(x)$. This objective function should be minimized while taking into account the restrictions of the job. Although this technique may be easy for users to understand and use, it is important to establish the weights that the decision maker (DM) will assign in advance, as they have a considerable

influence on the final result. The utilization of the weighted sum technique yields the subsequent normalized single-objective optimization problem.

$$Minimize\ Z = W_1Z_1 + W_2Z_2 + \cdots + W_TZ_T$$

Subject to

$$\sum_{j=1}^{n} X_{ij} = a_i,$$

$$\sum_{i=1}^{m} X_{ij} = b_j,$$

$$X_{ij} \geq 0, i = 1,2 \ldots, m\ \ j = 1,2, \ldots, n$$

The weights $W_t, t = 1,2, \ldots, T$ that are assigned to the objective function must satisfy the following conditions. $\sum_{t=1}^{T} W_t = 1, 0 \leq W_t \leq 1, t = 1,2, \ldots, T.$ By employing the aforementioned methodology, different solution points are obtained for different weights, effectively reflecting the preferences of the decision-maker. This strategy loses its effectiveness when the decision maker lacks knowledge or comprehension of the individual's preferences.

The Weighted Sum Method is a decision-making technique used in multi-criteria decision analysis. It involves assigning weights to the criteria based on their relative importance and then calculating a weighted sum for each alternative to determine the overall best choice.

Peter C. Fishburn introduced the weighted sum approach in 1967 in his article titled "Additive Utilities with Incomplete Product Set: Applications to Priorities and Assignments" (Fishburn, 1967). Fishburn discovered that the weighted sum approach is a fast and effective method for decision-making that takes into account multiple criteria.

4.2.Weibull distribution

The probability density function of the Weibull distribution is defined by the known parameters η, β and χ.

$$\mathrm{f}(s, \eta, \beta, \chi) = \frac{\eta}{\beta}\left(\frac{s-\chi}{\beta}\right)^{\eta-1} . e^{-\left[\left(\frac{s-\chi}{\beta}\right)^{\eta}\right]}$$

and

$$F(s) = 1 - e^{-\left[\left(\frac{s-\chi}{\beta}\right)^{\eta}\right]}$$

Where $f(s) \geq 0, s \geq 0 \ \ or \ \chi, \eta > 0, \beta > 0, -\infty < \chi < \infty$. The shape parameter, referred to as the Weibull slope, the scale parameter, and the location parameter, denoted as η, β, and χ, respectively, should be acknowledged.

The Weibull distribution (Krishnamoorthy (2006)) is a continuous probability distribution that can be employed to represent several variables, including the time before failure, the duration between events, and extreme values. The concept was initially comprehensively elucidated in 1939 by the Swedish mathematician Waloddi Weibull, who subsequently lent his name to it. The Weibull distribution is utilized in various fields such as Reliability engineering, Life data analysis, Engineering, Biology, Economics, and more.

The Weibull distribution has three parameters:

- Shape parameter (η): This parameter controls the appearance of the distribution. An exponential distribution is generated by a shape parameter of 1, but a shape parameter greater than 1 yields a distribution with an elongated tail.
- Scale parameter (β): This parameter determines the scale of the distribution. The point reflects the greatest value of the probability density function.
- Location parameter (χ): This parameter determines the position of the distribution. At the point where the cumulative distribution function reaches this value, it is equivalent to 0.5.

4.3. Newton's Divided Difference Interpolation (Agrawal and Ganesh (2020))

The methodology for addressing the transportation problem involves a wide range of solutions and varied aims. Firstly, the multi-choice parameter is modified using Newton's Divided Difference Interpolation.

Newton's Divided Difference Interpolating Polynomial for Multi-choice Parameters

The optimal choice is achieved by converting a multiple-choice parameter using Newton's Divided Difference Interpolation method. In order to define the interpolating polynomial for each option in a multi-choice parameter, it is necessary to specify an integer variable. In the current context, if there are s potential cost alternatives, the integer variables $w_{c_{ijk}}^{t}$ $(t = 0,1, \dots, s-1)$ are used.

Each multiple-choice parameter possesses a distinct divided difference that is determined by the provided choices. For the cost parameter $c_{ijk}(r) = (c_{ijk}^{1}(r), \dots, c_{ijk}^{s-1}(r))$ in Equation (7), the Newton's Divided Difference (NDD) interpolating polynomial is generated using the data given in Table 1, which lists the various orders of divided differences. The notation $c_{ijk} = \left(c_{ijk}^{1}, \dots, c_{ijk}^{s-1}\right)$ will be used to ensure universality.

Table 1: Divided difference (DD)

t	$w_{c_{ijk}}^{t}$	$f_{c_{ijk}}(w_{c_{ijk}}^{t})$	First DD	Second DD	$\cdots$ $(s-1)^{th}$ order DD
0	$w_{c_{ijk}}^{0}$	c_{ijk}^{1}			
			$f[w_{c_{ijk}}^{0}, w_{c_{ijk}}^{1}]$		
1	$w_{c_{ijk}}^{1}$	c_{ijk}^{2}		$f[w_{c_{ijk}}^{0}, w_{c_{ijk}}^{1}, w_{c_{ijk}}^{2}]$	
			$f[w_{c_{ijk}}^{1}, w_{c_{ijk}}^{2}]$		
2	$w_{c_{ijk}}^{2}$	c_{ijk}^{3}		$f[w_{c_{ijk}}^{1}, w_{c_{ijk}}^{2}, w_{c_{ijk}}^{3}]$	
			$f[w_{c_{ijk}}^{2}, w_{c_{ijk}}^{3}]$		$f[w_{c_{ijk}}^{0}, w_{c_{ijk}}^{1}, \dots, w_{c_{ijk}}^{s-1}]$
3	$w_{c_{ijk}}^{3}$	c_{ijk}^{4}		$\vdots$	
$\vdots$	$\vdots$	$\vdots$	$\vdots$		
$s-2$	$w_{c_{ijk}}^{s-2}$	c_{ijk}^{s-2}		$f[w_{c_{ijk}}^{s-3}, w_{c_{ijk}}^{s-2}, w_{c_{ijk}}^{s-1}]$	
			$f[w_{c_{ijk}}^{s-2}, w_{c_{ijk}}^{s-1}]$		
$s-1$	$w_{c_{ijk}}^{s-1}$	c_{ijk}^{s-1}			

Hence, the interpolating polynomial, considering these values, is

$$
\begin{aligned}
F_{c_{ijk}}^{r}\left(w_{c_{ijk}}\right) = f[w_{c_{ijk}}^{0}] &+ \left(w_{c_{ijk}} - w_{c_{ijk}}^{0}\right) f\left[w_{c_{ijk}}^{0}, w_{c_{ijk}}^{1}\right] \\
&+ \left(w_{c_{ijk}} - w_{c_{ijk}}^{0}\right)\left(w_{c_{ijk}} - w_{c_{ijk}}^{1}\right) f\left[w_{c_{ijk}}^{0}, w_{c_{ijk}}^{1}, w_{c_{ijk}}^{2}\right] \\
&+ \left(w_{c_{ijk}} - w_{c_{ijk}}^{0}\right)\left(w_{c_{ijk}} - w_{c_{ijk}}^{1}\right) \dots \left(w_{c_{ijk}} \right. \\
&\left. - w_{c_{ijk}}^{s-2}\right) f\left[w_{c_{ijk}}^{0}, w_{c_{ijk}}^{1}, \dots, w_{c_{ijk}}^{s-1}\right]
\end{aligned}
$$

Where,

$$f\left[w_{c_{ijk}}^{0}, \ldots, w_{c_{ijk}}^{t}\right] = \frac{f\left[w_{c_{ijk}}^{1}, \ldots, w_{c_{ijk}}^{t}\right] - f\left[w_{c_{ijk}}^{0}, \ldots, w_{c_{ijk}}^{t-1}\right]}{w_{c_{ijk}}^{t} - w_{c_{ijk}}^{0}}$$

The software replaces the multi-choice parameters with an interpolating polynomial.

4.4. Goal programming approach (Charnes and Cooper (1977))

The goal programming approach is a method used for making decisions when there are several objectives that may conflict with each other. It is especially beneficial when it is not feasible to optimize all objectives at the same time, and there is a requirement to balance and prioritize these objectives.

Now, let's expeditiously analyze the objective programming technique for solving MC-MOTP. Furthermore, let $d_t^+ = max(0, Z_t - g_t)$ denote positive deviations and $d_t^- = max(0, g_t - Z_t)$ denote negative deviations related to the t^{th} target of the objective function. The mathematical model is precisely defined in the following manner:

$$Minimize \ \sum_{t=1}^{T} W_t(d_t^+ + d_t^-)$$

Subject to

$$Z_t(x) - d_t^+ + d_t^- = y_t \ \ (t = 1,2, \ldots, T),$$
$$g_{t,min} \leq y_t \leq g_{t,max} \ \ (t = 1,2, \ldots, T),$$
$$d_t^+, d_t^- \geq 0 \ \ (t = 1,2, \ldots, T),$$

Between the upper bound $(g_{t,max})$ and lower bound $(g_{t,min})$, the t^{th} aspiration level, y_t, is a continuous variable.

5. Literature Review

Transportation problem is a special kind of Linear Programming Problem (LPP) in which goods are transported from a set of sources to a set of destinations subject to the supply and demand of the sources and destination respectively such that the total cost of transportation is minimized. It is also sometimes called as Hitchcock problem. Hitchcock (1941) established the fundamental principles of his groundbreaking research on the distribution of items from several origins to multiple locations. Koopmans (1949) coined the term "optimal utilization" to describe the efficient use of transportation infrastructure. The cost of the product may vary

from time to time, or it may depend upon the production of the product. Also, supply and demand may be dubious in nature due to the unavailability of the information of the shipped product. Due to these factors, and to deal with imprecise information Zadeh (1965) introduced the concept of fuzziness. Hierarchical optimization, or multi-level programming, deals with optimization problems that involve several decision-makers organized in a hierarchical structure (Lachhwani, K., 2015). Newton's Divided Difference a multi-choice fractional stochastic transport problem can be solved using interpolation by transforming it into a deterministic model (Agrawal, P. and Ganesh, T. 2020). New goal programming method handles "more/better" and "less/better" aspirations in complex decision problems (Chang, C.T. 2007). The authors propose an alternative method to formulate multi-choice aspiration levels in goal programming, which is easier to understand and can be solved by common linear programming packages (Chang, C.T. 2008). The MOTPs may be effectively addressed by treating it as a chance-constrained programming problem and using the global criteria technique and fuzzy goal programming methodology. This approach allows for the identification of optimal solutions within a realistic timeframe (Das, A. and Lee, G.M., 2021). The paper proposes a transformation technique to convert a multi-choice linear programming problem into a non-linear mixed integer programming model, enabling the attainment of optimal solutions using standard non-linear programming software (Biswal, M.P. and Acharya, S., 2009). The paper presents an alternative multi-choice goal programming formulation based on the conic scalarizing function, enabling the setting of multi-choice aspiration levels for each goal to achieve efficient solutions while minimizing auxiliary constraints and ensuring properly efficient outcomes (Ustun, O., 2012). Fuzzy programming finds a good compromise solution for complex multi-objective transportation problems, even better than traditional interactive methods (El-Wahed, W.F.A., 2001). The article presents an alternative approach for solving multi-level multi-objective linear fractional programming problems, demonstrating a comparatively less computational and efficient technique through a numerical example and comparative analysis with an earlier method (Joshi et al., 2022). The paper explores the transportation problem with a fractional objective function, offering a methodology for calculating total cost bounds directly and allowing for simultaneous changes in supply and demand values, with a numerical example provided to support the theory (Joshi, V.D., and Gupta, N., 2011). An interactive fuzzy goal programming method helps find preferred solutions for multi-objective transportation problems with uncertain data, considering decision-maker input while minimizing their involvement (El-Wahed et al. 2006). This study tackles multi-objective transportation problems with uncertain data and

fractional objectives, proposing three methods (weighted sum, fuzzy programming, and global criterion) to find compromise solutions (Saini, R. et al. 2023). This study uses weighted goal programming to solve multi-objective transportation problems, letting decision-makers prioritize objectives for a more customized solution (Nomani et al. 2016). A weighted approach lets decision-makers prioritize objectives in multi-objective transportation problems, finding a compromise solution tailored to their needs (Joshi et al. 2023). The study presents an expanded modi-method to solve a multi-index transportation problem involving three indices (Haley, K. B., 1962).

6. Description of topic

A Study "Development and Comparative Analysis of Multi-Objective Transportation Problem and its Variants" utilizes diverse mathematical strategies to manage the uncertainties and intricacies inherent in transportation networks. This methodology combines Nomani's Algorithm (2016), stochastic programming, goal programming, conic sacralization approach (Ustun (2012)), fuzzy approach, Newton divided difference, Lagrange's interpolating polynomial approach and uncertainty analysis to attain effective and resilient transportation solutions.

7. Objective of research

The objectives of the proposed study are as follows: -

- To study linear, solid, fully fuzzy, fractional and other multi-objective transportation problems.
- To study the solving techniques of linear, multi-choice and multi-objective transportation problems.
- To study fuzzy sets and systems in a linear and multi-objective programming problem.
- To study linear and fractional transportation problems.
- To study the solving techniques of linear, solid, fully fuzzy, fractional and other multi-objective transportation problems.
- To study multi-objective linear and fractional fixed-charged transportation problems with their solving techniques.
- Case study of multi-objective linear, solid, fractional and other transportation problems in real life.

8. Research methodology

We will discover new techniques for solving complex Multi-Objective Transportation problems, where the functions are multi-choice linear, solid, fractional and other Evolutionary Algorithms (Weighed Sum Method, Goal Programming, Conic Sacralization Approach, ε-Constraint Methods, Stochastic programming, fuzzy approach, and uncertainty analysis methodologies etc.) used to find the new techniques. We will discuss new techniques for using fuzzy theory in linear and fractional multi-objective programming problems.

9. Expected results of this research

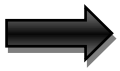
The expected outcomes of the proposed study are as follows: -

- We will develop methods for of multi-choice models for solving solid, uncertain transportation problems.
- It will help people to take decisions more economically and efficiently.
- More complex multi-objective transportation problems could be solved using goal programming techniques, fuzzy approach specifically tailored to solve with better efficiency.
- We can also formulate and solve some new real-life multi-objective transportation problems with some uncertainty criteria.

10. Work plan with time line

Research Activity (in months)	6	12	18	24	30	36	*
1. Course Work							
2. Research proposal submit							
3. Literature Survey							
4. Methodology Steps							
5. Quarterly Reports							
6. Communication of Papers							

7. **Preparation of Final Thesis**

8. **Submission of Thesis** 

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